Design Improvement of Aerospace Structures using Stochastic Simulation

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Abstract. The performance of aerospace structures depends on the selection of many design variables such as material thicknesses and properties, shapes, loads, associated boundary conditions, etc. To select the combination of variables for the best design, the sensitivity of the performance requirements with respect to the design variables must be determined. This can be achieved effectively using stochastic simulation that can accommodate a large number of variables, but with a limited number of FEM design evaluations. Sensitivity information facilitates trade-off studies as well as incorporation of uncertainties and tolerances.

This paper discusses the design improvement of aircraft structures using a Monte Carlo stochastic simulation technique, applied to a typical stringer-skin model. The variables from the results of the model were then stochastically simulated to obtain a range of input values. The results from the simulation enabled the selection of appropriate values for various input parameters to achieve an improved design.

Introduction. Creating innovative products and at the same time avoiding the pitfalls of reinventing known solutions or repeating mistakes is a major challenge for most companies [10]. Hence, applications that capture, store and re-use product evaluation and process knowledge are vital. Such products vertically integrate the knowledge and processes involved with building, testing and validating specific components and systems [10].

The benefits of adopting simulation techniques include a reduction in physical prototypes and early assessment of design configurations for organisations [10]. It is suggested in [10] that as more and more technology is embraced it is necessary to better manage engineering data and better integrate simulation processes with the product development process.

Background. Changability in dimensions, loads, material properties and associated boundary conditions leads to uncertainty in actual design performance [10]. Table 1 shows the natural scatter in material properties and loads.

<table>
<thead>
<tr>
<th>Material Characteristic</th>
<th>Coefficient of Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metallic Rupture</td>
<td>8 – 15%</td>
</tr>
<tr>
<td>Buckling</td>
<td>14%</td>
</tr>
<tr>
<td>Carbon Fibre Rupture</td>
<td>10 – 17%</td>
</tr>
<tr>
<td>Bonding Adhesive strength</td>
<td>12-16%</td>
</tr>
<tr>
<td>Metal/metal</td>
<td>8-13%</td>
</tr>
<tr>
<td>Honeycomb Tension</td>
<td>16%</td>
</tr>
<tr>
<td>Shear, compression</td>
<td>10%</td>
</tr>
<tr>
<td>Face wrinkling</td>
<td>8%</td>
</tr>
<tr>
<td>In-plane compression</td>
<td>15-20%</td>
</tr>
</tbody>
</table>

Table 1: Scatter in materials
According to Allen, 2004 [2], sources of uncertainty in real life structures arise from a number of factors such as:

- Material Properties
- Environmental Loads
- Boundary conditions
- Geometry imperfections (as shown in Fig. 1)
- Assembly imperfections
- Solver
- Computer (round – off, truncation, etc.)
- Engineer (choice of element type, mesh band – width, algorithm etc.)

Existing Methodologies. The article, Stochastic Software Manages Uncertainty, 2004 [3] highlights that historically uncertainty has been accounted for by incorporating safety factors in engineering and in finite element models, in particular. By employing this strategy, a stochastic problem is transformed into a deterministic one. The author in [3] further argues that doing so has resulted in over engineering and excessive costs. Moreover, by using safety factors, often some unknown is overlooked or unmodeled. If an unfortunate combination of factors and circumstances occurs, it could result in a catastrophic event, loss of life, or perhaps to an expensive law suit or recall [3]. This is because the main issue with safety factors is that they do not provide any measure of the real safety levels in the structure.

However, uncertainty can be accounted for without the use of safety factors and in the same way as it manifests itself in nature by exploiting advanced computational capabilities [3]. Thus by simulating reality, it is possible to manage uncertainty within product development with stochastic analysis. The author also contends that the benefits of doing this is based on a simple point – by incorporating uncertainty, models become realistic and, as engineers, it is essential to get as close as possible to reality before anything is brought to market [3]. In addition, since models that incorporate uncertainty become extremely realistic, it is possible to better understand and manage uncertainty.

Additionally, by incorporating realistic uncertainty into a Finite Element Model (FEM), simulations would be able to come up with acceptable, real world solutions instead of optimal, and sometimes fragile configurations [3]. Hence, design systems that better meet user-specified performance could be designed, and the overall product quality could be improved with less reliance on physical evaluation. Such a step would effectively manage risk associated with designs using Virtual Product Development techniques [3].

Principle of Complexity. As an engineering system becomes more complex and uncertain, it becomes more and more difficult to predict its behaviour until a limit is reached beyond which accuracy and significance become almost mutually exclusive [1]. Hence the Principle of Complexity suggests that high precision is incompatible with high complexity.

Furthermore, the Principle of Complexity also suggests that the greater the number of components that interact in a system, the less possible it is to make accurate conclusions about the system's performance and behaviour [3]. The author [3] further explains with the example that there can be 20 to 30% uncertainty or variation in materials or loading conditions. Adding more decimals or more computing horsepower does not change the outcome significantly. In this light, the software under consideration for stochastic simulation does not restrict users to a small number of variables, as in the case of design-of-experiment (DOE) methods. Three to four times the number of evaluations per variable is needed for DOE users. Thus, in order to get a response for 40 design variables, the solver would need to do 120 to 160 evaluations to get a response surface.
In contrast, stochastic simulations, handle thousands of design variables and yet only need only about 100 iterations.

**Realism versus Precision.** A model’s performance is very complex in reality. Hence, a model should be realistic foremost, not accurate. Marczyk (2004) asserts in [8] that “precision is not a characteristic of the universe, and the underlying physics that govern the universe is based on uncertainty principles”. The contention that art imitates nature, and since nature is not perfect, including of elements of uncertainty in computer models enhances the real-life capabilities of these models. MSC Robust Design© is a stochastic simulation tool, which carries this philosophy.

Randomising as many variables as practically possible in a finite element model is the way to go, says Marczyk (2004) [8]. He states that in most good designs only a small number of variables or features eventually drive the model. However, it is difficult to know which variables are important; besides, as the models become more complex, the chances that some unthought-of combination of uncertainties could have unexpected and potentially undesired effects become greater. Such unthought-of combinations can only be observed if the FE model incorporates many variables with tolerances. These permutations are represented as outliers, showing inconsistent behaviour, which can be directly converted into risk and liability. Therefore, it is important to anticipate their presence and fully comprehend the circumstances under which they arise. For this to happen, it is vital that the model is not biased, that is, the model must have the capabilities to actually produce these pathologies. For this to be possible, everything in the model must have a tolerance as reflected in reality. Most often this is not just 30-40 variables, but thousands of variables [8]. Doing so, the model can be made credible with claims about a model’s robustness.

Monte Carlo Simulation. Monte Carlo Simulation (MCS) is a process that repeatedly generates values for uncertain variables in a random manner in an attempt to simulate the conditions that may be experienced by the model in reality [4] (What is Monte Carlo Simulation?, 2005). To explain this concept further, the author in [4] draws an analogy between the random behaviour in games of chance and how Monte Carlo simulation randomly selects variable values to simulate a process. He says that if a person rolls a die, it is known that a 1, 2, 3, 4, 5, or 6 will result; however, the result of a particular roll is unknown. Similarly, variables have a known range of values but for a particular time or event, the value is unknown. Such variables include finite element characteristics, material properties, loads, and constraints for engineering applications.

Thus, a method that generates random numbers to solve a problem and the observation that fraction of these numbers obey some property is referred to as the Monte Carlo Method [1]. In summary, according to Perez Galan (2004) [11], the method is useful for

- Attaining global dependencies between variables of a design/model.
- Stochastic design improvement.
- Correlation with experimental data.

However, perhaps the strength of MCS is that the cost, hence the number of solver calls, is
independent of the number of variables in a problem [1]. An engineer can literally introduce thousands of stochastic variables in the design and only 100 solver calls would be needed to obtain correct results using MCS. As mentioned earlier, the more stochastic variables a model contains, the more realistic it is. Other advantages highlighted by Perez Galan (2004, p.8) [11] include:

- No restriction to type of analysis (linear, non-linear, multi-physics etc).
- Each solver call is independent of the other ones.
- Method is intrinsically parallel.
- Method convergence and robustness is independent of the number of variables.
- Statistical processing of results

**Stochastic Methodologies.** Stochastic implies the involvement of chance or probability. It is basically about Simulating Reality. Any analytical method that aims to imitate a real-life system, especially in instances where other analyses are mathematically too complex or too difficult to model is referred to as a simulation [4]. Nearly everything in nature is stochastic.

According to McPheeters, B. (2005) [9], stochastic (probabilistic) methods employ Monte Carlo techniques (mentioned earlier) to address uncertainty and variability in parameters such as geometry, material properties and loads to evaluate the conditions that are likely to eventuate when the structure is in service.

Allen (2004) [2] contends that engineering process is altered in a stochastic simulation. This is because there are

- No initial assumptions.
- Identifies what characteristics drive functionality.
- Can be a tool all engineers use.
- Provides direction and insight.
- Reduces risk through better engineering.

Thus, introducing of even minute tolerances in a FEM can result in unexpected behaviour of macroscopic quantities; this is referred to as butterfly effects and are quite common, says Marczyk (2004) [6]. Now, however, the most significant outcome of a stochastic simulation is more realistic, healthier, and more credible models [6]. Thus due to realistic and credible models, simulation could be used as a means of limiting liability and risk, and this cannot be done with surrogates of reality.

Stochastic simulation with a finite element model commences by specifying tolerances and scatter on all input variables used in the model (Marczyk, 2004) [6]. This process is known as randomisation. By including tolerances, the scatter in finite element models is increased significantly. Thickness, material properties, spring stiffness, beam cross-sections, loads, and imposed displacements having tolerances are examples of randomisation. The tolerances are defined by engineering limits and a particular distribution function [6]. At this stage in the simulation process, the engineer selects the desired outputs, such as stresses, frequencies, displacements, etc. Thereafter, the model is then executed a certain number of times (50-100 is typical), and each of the input variables within assigned tolerances is randomly changed. Based on Monte Carlo techniques, this is how stochastic simulation works [6].

The result obtained after simulation is referred to as a meta-model [6] and this appears like a constellation or cloud of points in 2D or 3D plots as shown in figure 2. Figure 2 shows a set of uncertain variable inputs, each represented by probability distributions, and the corresponding outputs. The model is executed approximately 100 times to stabilize the outputs of mean values and standard deviations. Hence, the randomisation process, though simple, has very important consequences. The more tolerances that are incorporated into a model, the

![Figure 2. System of stochastic simulation](source: Advances in Model Credibility by Addressing Variability (p.23) [2])
more realistic it becomes. Thus, Marczyk (2004) [6] suggests that even if a model has thousands of variables, not only those variables that are thought of as decisive or important should be randomised but all the variables should be randomised. Doing so, overcomes the limitations inherent to surrogate technique.

As mentioned above, a stochastic simulation results in a meta-model, which is essentially a matrix which has as many rows as the number of Monte Carlo samples + 1, and as many columns as the sum of input and output variables [1]. The meta-model can be projected on two or three dimensions yield ant-hill plots, or scatter plots.

McPheeters (2005) [9] recommends MSC Robust Design© as a tool for quick randomisation of an entire finite element model, wherein all the variables susceptible to tolerances can be randomised. He asserts that the software tool ranks variables by their correlation levels to visualise the variables. This is done by creating a variable matrix known as a Decision Map. By doing this, the variables are filtered to only show those variables that are significant to the model and therefore have significant effects. Such correlations represent a cause-effect relationship [9] and by understanding these relationships between variables good design decisions about the model can be made.

**Analysis Approach.** This project was done with the objective of improving the design of aerospace structures using stochastic simulation techniques. The project was in two stages. Firstly, a simple stringer-skin model had to be constructed using dimensions, loads and boundary conditions of an aircraft. This was followed by analysis of the model in MSC Robust Design©. The following sections explain how the model was constructed and analysed. In addition, the flowchart shown in figure 3 briefly outlines the approach taken for the analysis.

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**Figure 3. Flowchart depicting the analysis process**
Finite Element Modelling. Finite element modelling was used to mathematically approximate the behaviour of a structure with known physical and mechanical properties, says Di Cristoforo (1997) [6]. The model was sufficiently constrained and subjected to loads. This method was based on the discretisation of the structure into elements with known structural behaviour [6]. These elements are connected to one another by nodes at their vertices and are mathematically assembled according to their physical positions relative to each other. This results in a system of equations on the unknown nodal displacements. According to Bettoni (1994), such a system of equations is made up of the system stiffness matrix $[K]_s$, the unknown system displacement vector, $\{v\}_s$, and the system load vector, $\{P\}_s$. Hence, $[K]_s \{v\} = \{P\}_s$.

For the stringer-skin model used, hundreds of nodes and elements resulted and consequently hundreds of unknown displacements. As a system of equations of this size with so many unknowns is very difficult to practically solve, complex finite element analysis was used. Once the equations were solved for the unknown displacements, element stresses were determined by monitoring the stresses on the structure as a result of the applied loads. The model was constructed using the interactive graphic user interface of MSC Patran© and the finite element model was then analysed using MSC Nastran©.

It was assumed that modelling just one bay of the stringer-skin configuration would provide a fairly good indication of the response of the whole skin panel. The model was constructed in 2-D with 12 stringers as shown in the figure 4. The stringers were modelled as I-beams where the caps of the I-beam were modelled as bar elements, and the webs were modelled as shell elements.

The material properties identification enables the user to specify anisotropy of the material if it exists (Bettoni, 1994). This is an important consideration if the material has been rolled, extruded or forged. However, for this analysis, it was assumed that the structure was machined from an isotropic billet of aluminium, so the isotropic properties of the material were employed.

As the wing panel was made of aluminium, the material properties such as the Young’s modulus, shear modulus and Poisson’s ratio were specified. Along with these three values, other values such as the heat conductivity and material orientation could have been entered, depending on the type of analysis being undertaken. Since this analysis was done as a buckling mode analysis, the above values were sufficient for executing the analysis.

In the process of modelling, the stringer-skin model was effectively made up of 3 types of elements with different thicknesses. One property card was established for the skin, another for the stringer webs, while a third property was specified for the caps or flanges of the stringer.

The next step in the modelling process required the application of boundary conditions. Boundary conditions are applied to nodes, generally, by specifying which degrees of freedom are to be rigidly constrained. For example, constraining degrees of freedom 1, 2 and 3 will prevent translation in the X, Y and Z directions respectively, while constraining degrees of freedom 4, 5 and 6 would prevent rotation about the X, Y and Z axes respectively. Hence, there are many combinations that allow the user to represent other forms of constraint.

The stringer-skin model was constrained such that there was some fixed attachment at every location except the wing edge, which keeps the structure in place. As such, it was endeavoured to model the attachments as closely as possible to the real structure. In reality, the attachments do not constrain every degree of freedom. Assuming standard axes orientation, the X axis being along the structure while the Z axis being normal to the structure. Whilst translation in the Z-direction may be totally constrained, there may be some translation in the X and Y directions due to tolerances between the components of the structure. In addition, there may be some rotation in the structure as well.

For the stringer–skin model, the skin edge was fixed in all rotations and translation in the Z–direction, so the model was fixed in 3, 4, 5 and 6. The front and rear skin at the location of two ribs was fixed in translation in the Y and Z (hence 2 and 3) directions, while the free end of the skin was fixed in translation in the Z direction (that is, in 3). One corner of the skin on the fixed edge was also fixed as a hinge in all translations only, that is, in the X, Y and Z axes or in the 1, 2 and 3 directions. These have been shown in figure 4.
The final step in the construction of the finite element model was the definition of the applied loads. In this case, the load was assumed to be distributed along the leading edge of the wing panel as indicated by the red arrows shown in figure 4. Before applying the loads on the structure, they were first discretised. Discretisation of load involves splitting the distributed load into discrete forces that act on each node along its path (Di Cristoforo, 1997). Figure 5 shows the concept of how the load was discretised in this situation.

The results obtained from MSC Nastran© were graphically observed in Patran©. As such postprocessing of results is not required for the optimisation process. However by doing so, the analyst can gain insight into the behaviour of the structure by analysing pictures such as fringe plots of stresses or deformed shape plots. Details about normal stresses, shear stresses, principal stresses and Von Mises stresses of every element in the model is contained in element stresses file. By considering a fringe plot of Von Mises stresses, the analyst gets an indication of the reserve factor at every point on the model, since failure occurs when the Von Mises stress exceeds the material ultimate tensile stress.

Figure 6 shows a fringe plot of the maximum displacements as a result of buckling of the structure when the loads and boundary conditions are applied. The eigenvectors obtained as a result of the buckling analysis is shown in figures 7 and 8.

Once a working model was obtained in Patran©, the resultant model was imported into MSC Robust Design© for further analysis using stochastic simulation. This would determine the optimum range of values for various input parameters based on the results obtained.

Finite element model imports are easily done through MSC Robust Design©. Once a finite element model has been imported, the full model can be viewed in MSC Robust Design© through a project tree showing the finite elements, materials, loads, constraints and output. Clicking on the labels in the project tree enables easy randomisation of the properties of the finite elements, materials, loads and constraints [1]. All known tolerances and uncertainties are introduced into the FEM model. Default distribution functions have been established for quick randomisation. However, it is important to
note that the distribution function will generally not be known for most variables [1]. Hence, using the default would provide a more real picture of the functionality being analysed than a single deterministic value. Around 100 samples should typically be run in order to provide credible results.

The stringer-skin model was imported as a *.bdf file. Once the model, as shown in figure 9, was imported into MSC Robust Design®, a model health check was performed on the model. 

![Figure 9. Stringer-skin model in Robust Design®](image)

This was done to verify the overall behaviour of the system and to detect and understand abnormalities in the finite element model. Basically, such a step yields confidence in the model quality by considering factors such as stability and internal noise sources etc [1]. The author in [1] explains that when performing a health check, an arbitrary set of model parameters is randomly varied, involving solver-specific constants. This helps to detect model deficiencies or wrong modelling practise. The results have no statistical significance. It is suggested that as many variables as possible should be randomised in the health check, including those that are initially regarded as deterministic [1].

As the results of the health check performed on the model appeared to be reasonable, as there was no solver failure and the simulation ran to completion. Scatter in the model was present but it was reasonable and very few outliers were seen. Moreover the decision map obtained and the ant-hill plots that resulted did not indicate any abnormalities in the model that was imported from Patran® into the software. Hence, it was fine to proceed with further analysis of the structure.

A problem is ready to have a simulation run after the input variables (elements, materials, loads and constraints) have been randomised (distribution function defined) and the desired outputs have been selected [1]. This was the case with the stringer-skin model as well. A Coefficient of Variation of 5% was applied to the model. The number of solver runs was set to 100. As the first result of the simulation was obtained, the simulation design control tab gets populated with results as each simulation job is completed. The mean, minimum, maximum, most likely values and the coefficient of variation are shown for each observable output and design variable.

**Results.** A range of input and output results were obtained from running the simulation in MSC Robust Design®. Due to confidentiality reasons, these values (input, output and ranges) cannot be mentioned in this paper.

The meta model obtained as a result of the stochastic simulation contained a vast amount of information on how all of the variables influence the functionality of the function being simulated. The results of the meta model could be viewed from any combination of two or three input and output or output and output variables. The results of the simulation could be presented in several ways, thus transforming the meta model into meaningful information. The foundation is in the automatic creation of a decision map that shows the significant correlations between all the input and output variables and the relative levels of influence. This information was further transformed into pie charts and ant hill plots, reflecting the impact of simultaneous variations in input variables on the scatter of the performance. Such information is very useful in guiding the engineer to relax or increase the tolerances where necessary.

The decision map obtained after running the simulation of the stringer-skin model is shown in figure 10. Decision maps display the topology of information flow within the system [1]. As can be seen in the figure, it is possible for one to immediately see what is important, what is not, what drives the performance, how the variables relate to each other, global and local correlation levels and the potential to modify a design and design rules.
The figure above shows the relationships and the extent of correlation between the variables. The main input and output variables that influence the model have been shown and named in the decision map. The bright red connections indicate high correlation between the variables as in the case of buckling mode 1 and shell thickness, buckling mode 1 and stiffness mode 1 and others. Reasonably strong correlations, as depicted by the dull red connections, were also seen in the case of stiffness mode 1 and stiffness mode 2, stiffness mode 2 and shell thickness and a few other combinations of variables. However, less strong correlations were found to exist between stiffness mode 2 and mass mode 2, as shown by the dull blue connections on the left hand side of the screen.

Another way that was used to analyse the results was with the use of pie charts. By using pie charts it was possible to see how the scatter (tolerances) in all the inputs influences the scatter of a certain output. Thus for any output variable, a pie chart will display the relative importance of all variables, while subjected to tolerances. It is important to note that these influences are not sensitivities, but Spearman correlations (essentially a ranking of the variables) to the scatter of the output variable being considered [1].

A pie chart for the generalised stiffness mode 1 is shown in figure 11. As seen in the figure, the single most influential input parameter was shell thickness. The importance of the other input parameters is also shown in the pie chart according to the extent to which the variable is important for a specific output. As such, in all the inputs there was a significant contribution of a number of smaller inputs, but each of those inputs taken individually was not significant enough to be shown as a separate input variable in the pie chart. The results are also consistent with the Euler buckling equations, which indicate that the thickness of the structure has the maximum influence on the manner in which the structure responds as a result of the application of loads and boundary conditions.

Another method to graphically represent results of the stochastic simulation in MSC Robust Design is by using ant-hill plots. Like the other methods of displaying results, ant-hill plots can be generated by clicking on the nodes. They can be generated in 2D (using 2 variables) or 3D (using 3 variables) [1]. Depending on the results needed, ant-hill plots can be very useful. The orientation and shape of the plot conveys significant information about the correlation of variables as shown in figure 12.

The ant-hill plot shows a strong linear correlation of 0.916 and a non-linear correlation of 0.922 for the input output variables. The plot also indicates that there are no major bifurcations or outliers as a result of the stochastic simulation, and hence provides useful information to the engineer in the decision making process.
Conclusion. In the process simulating this model, it was found that by importing a finite element model into a software such as MSC Robust Design©, it was possible to determine the key variables that influence the performance of the model. The stochastic simulation process employed as the basis of the software made it possible to analyse results to establish which variables were important for the structure in real life. The results also enabled the authors to determine that no detrimental combination of variables would result, causing significant damage to the structure. This was verified from the fact that there were no significant outliers or bifurcations that emerged from the graphical representation of the results.

Furthermore, the shell thickness was found to be the most important input variable that influenced the results of the model. It was also deduced that a stochastic simulation software like MSC Robust Design© is a powerful tool that can help in optimising the structure provided the inputs for the software were defined in the MSC Patran©. Thus, in the case of this project, it is possible to optimise the model for buckling, stiffness and mass of the structure.

Based on the results obtained for this project, it can be concluded that MSC Robust Design© is a powerful tool in improving the design of aerospace structures using stochastic simulation. By employing Monte Carlo methods to simulate real world conditions, it is possible to get a wholistic picture of the performance of a structure in its actual environment. By identifying the abnormalities such as bifurcations and outliers, it is possible for the engineer to analyse and design for non-intuitive scenarios resulting from a combination of variables which could be critical in the life of the structure.

Future research could be carried out to determine methods by which input parameters could be specified in MSC Robust Design© rather than being dependent on the inputs provided from the Nastran© results. One option would be to incorporate that capability in the software itself; however, though difficult, it would also be possible to use other user defined methods to achieve such an objective.

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